

The Central Error of MYRON W. EVANS' ECE Theory - a Type Mismatch

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Abstract. In Sect.1 we give a sketch of the basics of spacetime manifolds. Namely the tetrad coefficients $q_{\mathbf{a}}^{\mu}$, are introduced which M.W.EVANS believes to be an essential tool of argumentation leading far beyond the limitations of General Relativity because of giving the opportunity of modelling several other force fields of modern physics in addition to gravitation. However, as we shall see in Sect.2, the main errors of that “theory” are *invalid* field definitions: They are simply invalid and therefore useless due to *type mismatch*. This is caused by M.W.EVANS' **bad habit** of suppressing seemingly unimportant indices. **There is no possibility of removing the tetrad indices \mathbf{a}, \mathbf{b} from M.W.EVANS' field theory, i.e. the ECE Theory cannot be repaired.** In Sect.3 M.W.EVANS' concept of a non-Minkowskian spacetime manifold [1; Sect.2],[2; Chap.3.2], is shown to be erroneous. In Section 4 another erroneous claim of [1; Sect.3], [2; Chap.3.3] is discussed.

The following review of M. W. EVANS' **E**instein **C**artan **E**vans field theory refers to M. W. EVANS' FoPL article [1]. About one year later he took the article over into his book [2] without essential changes. The labels below of type (3.·)/(·) refer to [2]/[1] respectively.

1. What M. W. EVANS should have given first: A clear description of his basic assumptions

M.W.EVANS constructs his spacetime by a dubious *alternative* method to be discussed in Sect.3 . Here we sketch the *usual* method of constructing the 4-dimensional spacetime manifold \mathcal{M} . The tangent spaces \mathbf{T}_P at the points P of \mathcal{M} are spanned by the tangential basis vectors $\mathbf{e}_{\mu} = \partial_{\mu}$ ($\mu = 0, 1, 2, 3$) at the respective points P of \mathcal{M} .

There is a pseudo-metric defined at the points P of \mathcal{M} as a bilinear function $g : \mathbf{T}_P \times \mathbf{T}_P \rightarrow \mathbf{R}$. Therefore we can define the matrix $(g_{\mu\nu})$ by

$$(1.1) \quad g_{\mu\nu} := g(\mathbf{e}_{\mu}, \mathbf{e}_{\nu}),$$

which is assumed to be of Lorentzian signature, i.e. there exist vectors $\mathbf{e}_{\mathbf{a}}$ ($\mathbf{a} = \mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}$) in each \mathbf{T}_P such that we have $g(\mathbf{e}_{\mathbf{a}}, \mathbf{e}_{\mathbf{b}}) = \eta_{\mathbf{ab}}$ where the matrix $(\eta_{\mathbf{ab}})$ is the Minkowskian diagonal matrix $diag(-1, +1, +1, +1)$. We say also the signature of the

metric $(g_{\mu\nu})$ is supposed to be Lorentzian, i.e. $(-, +, +, +)$.

A linear transform $L : \mathbf{T}_P \rightarrow \mathbf{T}_P$ that fulfils $g(L\mathbf{e}_a, L\mathbf{e}_b) = g(\mathbf{e}_a, \mathbf{e}_b)$ is called a (local) Lorentz transform. The Lorentz transforms of \mathbf{T}_P constitute the well-known (local) Lorentz group. All Lorentz-transforms have the property $g(L\mathbf{V}, L\mathbf{W}) = g(\mathbf{V}, \mathbf{W})$ for arbitrary vectors \mathbf{V}, \mathbf{W} in \mathbf{T}_P .

Each set of orthonormalized vectors \mathbf{e}_a ($\mathbf{a} = \mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}$), in \mathbf{T}_P is called a *tetrad* at the point P . We assume that a certain tetrad being chosen at each \mathbf{T}_P of the manifold \mathcal{M} . Then we have linear representations of the coordinate basis vectors $\mathbf{e}_\mu = \partial_\mu$ ($\mu = 0, 1, 2, 3$) by the tetrad vectors at P :

$$(1.2) \quad \mathbf{e}_\mu = q_\mu^{\mathbf{a}} \mathbf{e}_a.$$

From (1.1) and (1.2) we obtain due to the bilinearity of $g(\cdot, \cdot)$

$$(1.3) \quad g_{\mu\nu} = g(\mathbf{e}_\mu, \mathbf{e}_\nu) = q_\mu^{\mathbf{a}} q_\nu^{\mathbf{b}} g(\mathbf{e}_a, \mathbf{e}_b) = q_\mu^{\mathbf{a}} q_\nu^{\mathbf{b}} \eta_{\mathbf{ab}}.$$

The matrix $(g_{\mu\nu})$ is symmetric therefore. And more generally also $g(\mathbf{V}, \mathbf{W}) = g(\mathbf{W}, \mathbf{V})$ for arbitrary vectors \mathbf{V}, \mathbf{W} of \mathbf{T}_P . In addition, the multiplication theorem for determinants yields the matrix $(g_{\mu\nu})$ to be nonsingular.

A (non-Riemannian) linear connection is supposed, i.e. we have *covariant* derivatives D_μ in direction of \mathbf{e}_μ given by

$$(1.4) \quad D_\mu F := \partial_\mu F$$

for functions F ($= (0, 0)$ -tensors), while a $(1, 0)$ -tensor F^ν has the derivative

$$(1.5) \quad D_\mu F^\nu := \partial_\mu F^\nu + \Gamma_{\mu \rho}^\nu F^\rho$$

and for a $(0, 1)$ -tensor F_ν we have

$$(1.6) \quad D_\mu F_\nu := \partial_\mu F_\nu - \Gamma_{\mu \nu}^\rho F_\rho.$$

For coordinate dependent quantities the connection causes the additional terms in Eqns.(1.5-1.6) with the coefficients $\Gamma_{\mu \nu}^\rho$.

By the analogue way the connection gives rise to additional terms with coefficients $\omega_{\mu \mathbf{b}}^{\mathbf{a}}$ for the covariant derivatives of tetrad dependent quantities, namely

$$(1.7) \quad D_\mu F^{\mathbf{a}} := \partial_\mu F^{\mathbf{a}} + \omega_{\mu \mathbf{b}}^{\mathbf{a}} F^{\mathbf{b}}$$

and

$$(1.8) \quad D_\mu F_{\mathbf{a}} := \partial_\mu F_{\mathbf{a}} - \omega_{\mu \mathbf{a}}^{\mathbf{b}} F_{\mathbf{b}}.$$

2. M.W. EVANS' Generally Covariant Field Equation

M.W. EVANS starts with Einstein's Field equation

$$(2.1) \quad R^{\mu\nu} - \frac{1}{2}R g^{\mu\nu} = T^{\mu\nu}$$

which is "multiplied" by $q_\nu^{\mathbf{b}}$ $\eta_{\mathbf{ab}}$ to obtain

$$(2.2) \quad R_{\mathbf{a}}^\mu - \frac{1}{2}R q_{\mathbf{a}}^\mu = T_{\mathbf{a}}^\mu.$$

Here he suppresses the tetrad index \mathbf{a} :

Quote from [2]/[1]

$$(3.18)/(16) \quad R^\mu - \frac{1}{2}R q^\mu = T^\mu$$

He now “wedges” that by q'_b to obtain

$$(2.3) \quad R^\mu_a \wedge q'_b - \frac{1}{2}R q^\mu_a \wedge q'_b = T^\mu_a \wedge q'_b.$$

Here he suppresses the tetrad indices \mathbf{a}, \mathbf{b} again:

Quote from [2]/[1]:

$$(3.25)/(23) \quad R^\mu \wedge q^\nu - \frac{1}{2}R q^\mu \wedge q^\nu = T^\mu \wedge q^\nu$$

Remark

The wedge product used by M.W. EVANS here is the wedge product of vectors $\mathbf{A} = A^\mu \mathbf{e}_\mu$:

$$\mathbf{A} \wedge \mathbf{B} = \frac{1}{2}(A^\mu B^\nu - A^\nu B^\mu) \mathbf{e}_\mu \wedge \mathbf{e}_\nu$$

written in short hand as

$$A^\mu \wedge B^\nu := \frac{1}{2}(A^\mu B^\nu - A^\nu B^\mu). \quad \blacksquare$$

M.W. EVANS remarks the term $R^\mu \wedge q^\nu$ being antisymmetric like the electromagnetic stress tensor $G^{\mu\nu}$. Hence he feels encouraged to try the following ansatz

Quote from [2]/[1]:

$$(3.29)/(27) \quad G^{\mu\nu} = G^{(0)}(R^{\mu\nu(A)} - \frac{1}{2}R q^{\mu\nu(A)})$$

where

$$(3.26-27)/(24-25) \quad R^{\mu\nu(A)} = R^\mu \wedge q^\nu, \quad q^{\mu\nu(A)} = q^\mu \wedge q^\nu.$$

Thus, M.W. EVANS' ansatz (3.29)/(27) with written tetrad indices is

$$(2.4) \quad G^{\mu\nu} = G^{(0)}(R^\mu_a \wedge q'_b - \frac{1}{2}R q^\mu_a \wedge q'_b).$$

However, by comparing the left hand side and the right hand side it is evident that the ansatz cannot be correct due to *type mismatch*: The tetrad indices \mathbf{a} and \mathbf{b} are not available at the left hand side.

M.W. EVANS' field ansatz (3.29)/(27) is unjustified due to type mismatch.

The tetrad indices \mathbf{a}, \mathbf{b} must be removed *legally*. The only way to do so is to sum over \mathbf{a}, \mathbf{b} with some weight factors $\chi^{\mathbf{ab}}$, i.e. to insert a factor $\chi^{\mathbf{ab}}$ on the right hand side of (3.29)/(27), at (2.4) in our detailed representation. Our first choice for $\chi^{\mathbf{ab}}$ is the Minkowskian $\eta^{\mathbf{ab}}$. However, then the right hand side of (3.29)/(27) vanishes since we have

$$(2.5) \quad q^\mu_a \wedge q'_b \eta^{\mathbf{ab}} = q^\mu_a q'_b \eta^{\mathbf{ab}} - q'_a q^\mu_b \eta^{\mathbf{ab}} = g^{\mu\nu} - g^{\nu\mu} = 0$$

and

$$(2.6) \quad R_{\mathbf{a}}^{\mu} \wedge q_{\mathbf{b}}^{\nu} \eta^{\mathbf{ab}} = R_{\mathbf{a}}^{\mu} q_{\mathbf{b}}^{\nu} \eta^{\mathbf{ab}} - R_{\mathbf{a}}^{\nu} q_{\mathbf{b}}^{\mu} \eta^{\mathbf{ab}} = R^{\mu\nu} - R^{\nu\mu} = 0$$

due to the symmetry of the metric tensor $g^{\mu\nu}$ and of the Ricci tensor $R^{\mu\nu}$ [4; (3.91)].

One could try to find a matrix ($\chi^{\mathbf{ab}}$) different from the Minkowskian to remove the indices \mathbf{a}, \mathbf{b} from equations (3.25-29)/(23-27). That matrix should not depend on the special tetrad under consideration i.e. be invariant under arbitrary Lorentz transforms L :

$$(2.7) \quad L_{\mathbf{c}}^{\mathbf{a}} \chi^{\mathbf{cd}} L_{\mathbf{d}}^{\mathbf{b}} = \chi^{\mathbf{ab}} \quad \text{where} \quad L \mathbf{e}_{\mathbf{a}} =: L_{\mathbf{a}}^{\mathbf{b}} \mathbf{e}_{\mathbf{b}}.$$

However, due to the definition of the Lorentz transforms the matrices λ ($\eta^{\mathbf{ab}}$) with some factor λ are the only matrices with that property.

Therefore we may conclude that only a trivial **zero** em-field $G^{\mu\nu}$ can fulfil the *corrected* M.W. EVANS field ansatz.

The correction of M.W. EVANS' antisymmetric field ansatz (3.29)/(27) yields the trivial zero case merely and is irreparably therefore.

3. Further Remarks

The following remarks concern M.W. EVANS' idea of the spacetime manifold as represented in his [2; Chap.3.2]/[1; Sec.2].

He starts with a curvilinear parameter representation $\mathbf{r} = \mathbf{r}(u_1, u_2, u_3)$ in a space the property of which is not explicitly described but turns out to be an Euclidean \mathbf{R}^3 due to the Eqns.(3.10)/(8) below.

Quote from [2]/[1]:

Restrict attention initially to three non-Euclidean space dimensions. The set of curvilinear coordinates is defined as (u_1, u_2, u_3) , where the functions are single valued and continuously differentiable, and where there is a one to one relation between (u_1, u_2, u_3) and the Cartesian coordinates. The position vector is $\mathbf{r}(u_1, u_2, u_3)$, and the arc length is the modulus of the infinitesimal displacement vector:

$$(3.7)/(5) \quad ds = |d\mathbf{r}| = \left| \frac{\partial \mathbf{r}}{\partial u_1} du_1 + \frac{\partial \mathbf{r}}{\partial u_2} du_2 + \frac{\partial \mathbf{r}}{\partial u_3} du_3 \right|.$$

The metric coefficients are $\frac{\partial \mathbf{r}}{\partial u_i}$, and the scale factors are:

$$(3.8)/(6) \quad h_i = \left| \frac{\partial \mathbf{r}}{\partial u_i} \right|.$$

The unit vectors are

$$(3.9)/(7) \quad \mathbf{e}_i = \frac{1}{h_i} \frac{\partial \mathbf{r}}{\partial u_i}$$

and form the $O(3)$ symmetry cyclic relations:

$$(3.10)/(8) \quad \mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3, \quad \mathbf{e}_2 \times \mathbf{e}_3 = \mathbf{e}_1, \quad \mathbf{e}_3 \times \mathbf{e}_1 = \mathbf{e}_2,$$

where $O(3)$ is the rotation group of three dimensional space [3-8]. The curvilinear coordinates are orthogonal if:

$$(3.11)/(9) \quad \mathbf{e}_1 \cdot \mathbf{e}_2 = 0, \quad \mathbf{e}_2 \cdot \mathbf{e}_3 = 0, \quad \mathbf{e}_3 \cdot \mathbf{e}_1 = 0.$$

The symmetric metric tensor is then defined through the line element, a one form of differential geometry

NO! A symmetric TWO-form :

$$(3.12)/(10) \quad \omega_1 = ds^2 = q^{ij(S)} du_i du_j,$$

and the anti-symmetric metric tensor through the area element, a two form of differential geometry:

$$(3.13)/(11) \quad \omega_2 = dA = -\frac{1}{2} q^{ij(A)} du_i \wedge du_j.$$

These results generalize as follows to the four dimensions of any non-Euclidean space-time:

$$(3.14)/(12) \quad \omega_1 = ds^2 = q^{\mu\nu(S)} du_\mu du_\nu,$$

$$(3.15)/(13) \quad \omega_2 = {}^* \omega_1 = dA = -\frac{1}{2} q^{\mu\nu(A)} du_\mu \wedge du_\nu. \quad \boxed{\text{WRONG!}}$$

In differential geometry the element du_σ is dual to the wedge product $du_\mu \wedge du_\nu$.

WRONG! NOT in 4-D.

The symmetric metric tensor is:

$$(3.16)/(14) \quad q^{\mu\nu(S)} = \begin{bmatrix} h_0^2 & h_0 h_1 & h_0 h_2 & h_0 h_3 \\ h_1 h_0 & h_1^2 & h_1 h_2 & h_1 h_3 \\ h_2 h_0 & h_2 h_1 & h_2^2 & h_2 h_3 \\ h_3 h_0 & h_3 h_1 & h_3 h_2 & h_3^2 \end{bmatrix}$$

and the anti-symmetric metric tensor is:

$$(3.17)/(15) \quad q^{\mu\nu(A)} = \begin{bmatrix} 0 & -h_0 h_1 & -h_0 h_2 & -h_0 h_3 \\ h_1 h_0 & 0 & -h_1 h_2 & h_1 h_3 \\ h_2 h_0 & h_2 h_1 & 0 & -h_2 h_3 \\ h_3 h_0 & -h_3 h_1 & h_3 h_2 & 0 \end{bmatrix}$$

(End of quote)

The symmetric metric (3.16)/(14) cannot be correct since having a *vanishing* determinant: All line vectors are parallel. The reason is that the author M.W. EVANS has forgotten to insert the scalar products of his basis vectors. A similiar argument holds for Equ.(3.17) being dubious.

However, even if one avoids all possibilities mentioned above of going astray M.W. EVANS' method has *one crucial shortcoming*: The metric definable by that method. As follows from (3.7)/(5) we have $ds^2 \geq 0$, i.e. the metric is *positive definite*. That is a heritage of M.W. EVANS' construction of spacetime as an embedding into a real Euclidian space (defining the metric by (3.7)/(5)) that one cannot get rid off.

M.W. EVANS' construction cannot yield a spacetime with local Minkowskian i.e. indefinite metric.

That was the reason why we sketched a correct method of constructing the spacetime manifold of General Relativity at the beginning of this article in Sect.1. M.W. EVANS' alternative method of [2; Chap.3.2]/[1; Chap.2] is useless.

4. A Remark on [2; Chap.3.4]/[1; Sect.4]

With

Quote from [2]/[1]

$$(3.2)/(43) \quad R^\mu = \alpha q^\mu$$

claims *proportionality* between the tensors $R_{\mathbf{a}}^\mu$ and $q_{\mathbf{a}}^\mu$:

$$(4.1) \quad R_{\mathbf{a}}^\mu = \alpha q_{\mathbf{a}}^\mu.$$

However, there is no proof in [2; Chap.3.4]/[1; Sect.4] available. Indeed, if we assume (4.1) then we obtain the curvature

$$(4.2) \quad R = R^{\mu\nu} g_{\mu\nu} = (R_{\mathbf{a}}^\mu \eta^{\mathbf{ab}} q_{\mathbf{b}}^\nu) g_{\mu\nu} = R_{\mathbf{a}}^\mu q_{\mu}^{\mathbf{a}} = \alpha q_{\mathbf{a}}^\mu q_{\mu}^{\mathbf{a}} = 4 \alpha,$$

but the equation $R_{\mathbf{a}}^\mu q_{\mu}^{\mathbf{a}} = \alpha q_{\mathbf{a}}^\mu q_{\mu}^{\mathbf{a}}$ may have other solutions than (4.1). Hence there is no way from (4.2) back to (4.1).

The considerations of [2; Chap.3.4]/[1; Sect.4] may be based on a logical flaw.

References

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